Grade 4 Subtractive Division Algorithm

4.N.7	
Demonstrate an understanding of division (1-digit divisor and up to 2-digit dividend) to solve problems by • using personal strategies for dividing with or without concrete materials • estimating quotients • relating division to multiplication	(It is not intended that remainders be expressed as decimals or fractions.) 1. Solve a division problem without a remainder using arrays or base-10 materials. (**see comment in SET SCENE**) 2. Solve a division problem with a remainder using arrays or base-10 materials. (**see comment in SET SCENE**) 3. Solve a division problem using a personal strategy, and record the process. (**see Clarification of outcome**) 4. Create and solve a word problem involving a 1- or 2-digit dividend. 5. Estimate a quotient using a personal strategy (e.g., 86 ÷ 4 is close to 80 ÷ 4 or close to 80 ÷ 5).

Clarification of the outcome:

- ✦ While personal strategies are useful to begin the development of a division algorithm, the teacher should move towards an efficient algorithm that students understand. One appropriate algorithm involves subtracting groups of the divisor and keeping track of how many groups are subtracted. The teacher should move beyond clumsy personal strategies that students may "invent" and move towards a more elegant and efficient method for dividing. The crucial point is that students need to understand this algorithm.
- ✦ There are many writing styles for a division algorithm. Two are developed in this lesson. [One should not confuse 'writing style' with 'what is going on' in an algorithm.]

Required close-to-at-hand prior knowledge:

- ✤ Subtraction skills (automaticity of facts & subtracting 2-digit numbers).
- ✤ Addition skills (automaticity of facts & adding 2-digit numbers).
- ✤ Multiplication skills (automaticity of facts & multiplying 1-digit x 2-digit numbers).
- ✤ Understanding division means splitting up into equal groups
- ✤ Place value to the hundreds place

SET SCENE stage

The problem task to present to students:

Organize students into groups.

- ★ Ask each group to list two examples of when division is needed to solve a problem.
- ★ Ask each group to list two reasons why it might be important to learn how to divide, using pencil and paper.

Comments:

The SET SCENE approach here is a direct way of attending to the matter of why should any division algorithm be learned in these days of calculators, computers, and cell phones. Sometimes a direct approach is best. It brings fuzzy thoughts and negative emotions out into the open where they can be more readily addressed.

While the Manitoba curriculum guide suggests using base-ten materials in two of the achievement indicators, the reality is that these materials are clunky to work with for division. Inevitably (unless questions are carefully crafted so that the number of ten-pieces is exactly divisible by the divisor), a ten-piece will need to be traded (broken up) for ten one-pieces. It is less distracting and just as effective to simply use counters instead.

DEVELOP stage

Activity 1: Revisits SET SCENE.

- ✤ Ask each group for its two examples of when division is needed. Record distinct examples on the board. Ensure students realize division is used to solve problems.
- ✤ Ask each group for its two reasons for why it might be important to learn how to divide, using pencil and paper. Record distinct reasons on the board. [*Likely to concern power failures, inoperative batteries, and not feeling dumb*] Ensure students realize that even in these days of electronic devices, pencil and paper methods are still needed.

Activity 2: Addresses achievement indicators 1 and 3.

Present a story problem (2-digit number less than 40). For example:

20 candies are shared equally so that each person gets 4 candies. How many people get a share of the candies?

- ✦ Ask students to solve the problem any way they want. Provide counters (see comment in SET SCENE).
- ✦ Discuss students' solutions and methods. Discuss how the meaning of division is involved in the method used.

Activity 3: Addresses achievement indicators 2 and 3.

Provide three problems (2-digit number less than 40) that involve remainders where remainders have to be treated in real way (E.G: *Sam wants to share all of 38 treats with his 5 buddies equally. Sam also gets an equal share. How many treats do Sam and his buddies get each? What happens to the extra treats?* E.G: *35 students are going on a school trip by van. Each van can hold 6 students. How many vans are needed?*). Provide counters as needed. Ask students to solve the problems and record the process they use. Discuss solutions and methods.

Activity 4: Addresses achievement indicator 5.

Provide 2-digit (less than 40) divided by 1-digit questions (e.g. 38 ÷ 3). Ask students to estimate the answer. Discuss their methods and results. [E.G. for 38 ÷ 3, students might think 40 ÷ 4, so the answer is about 10.]

Note:

Many students will have been forming equal groups and/or thinking subtraction to obtain answers to the division questions. Their recording of the process likely varied widely. It might be useful to develop a recording process that is consistent (the subtractive algorithm). Doing so may help some students better understand how to do division.

Activity 5: Addresses achievement indicator 1 and 3.

Present a division question (2-digit number less than 50) having no remainder (e.g.: 24 ÷ 4). Ask students to obtain an answer using counters. Ask for and discuss briefly methods that involved forming equal groups.

Select one of the methods. Suppose that the student made groups of 4 and then counted the number of groups formed. Show students a paper-and-pencil writing style for the student's thinking (see sample).

Discuss the relationship between the work with the counters and the paper-and-pencil work. Ensure that students understand that (for example):

- '- 4' is forming a group of 4 and removing it from the pile,
- '20' is what is left after a group of 4 is removed,
- '1' is keeping track of each group of 4 removed,
- '6' is the total number of groups of 4 removed.

Have students check that 24 ÷ 4 is 6 by reversing the thinking (make 6 groups of 4 to obtain 24).

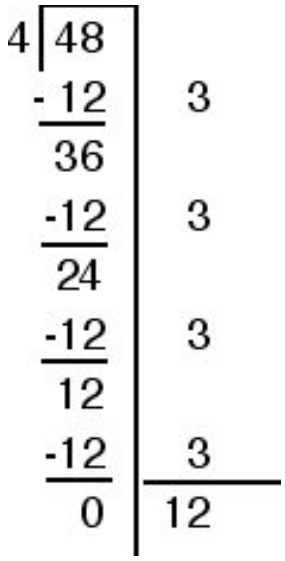
```
4 | 24
  - 4 | 1
  20
  - 4 | 1
  16
  - 4 | 1
  12
  - 4 | 1
  8
  - 4 | 1
  4
  4 | 1
  0 | 6
```

Provide another division question (2-digit number less than 50; e.g. 42 ÷ 6). Ask students to obtain an answer by using forming groups of 6 and subtraction thinking and the new writing style. Ask for and discuss student responses. The writing style should look as shown here.

```
6 | 42
  - 6 | 1
  36
  - 6 | 1
  30
  - 6 | 1
  24
  - 6 | 1
  18
  - 6 | 1
  12
  - 6 | 1
  6
  - 6 | 1
  0 | 7
```

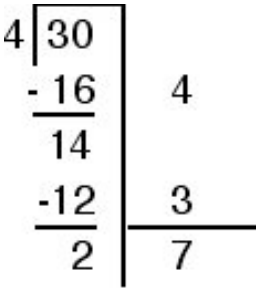
Activity 6: Addresses achievement indicator 3 (no remainder).

Provide students with four 2-digit (less than 50) divided by 1-digit division questions having no remainders (48 ÷ 4, 35 ÷ 5, 38 ÷ 2, 30 ÷ 6). Ask them to use a paper-and-pencil method to figure out the answer. Suggest that they could remove larger counts of groups (for example, instead of removing 1 group of 4, remove 3 groups of 4). Discuss solutions and recording method. Ensure they begin to remove larger groups of counts. See the example here. It shows the answer is 12, with a reminder of 0.



Activity 7: Addresses achievement indicator 3 (with remainder).

Provide students with three 2-digit (less than 50) divided by 1-digit division questions that have a remainder (e.g. 30 ÷ 4, 31 ÷ 5, 27 ÷ 2). Ask them to use a paper-and-pencil method to figure out the answer. Ask them to remove larger counts of groups (for example, instead of removing 1 group of 4, remove 5 groups of 4). Discuss their solutions. Ensure students remove larger groups of counts. See the example here. It shows the answer is 7, with a reminder of 2.



[Notice the obvious place to write the remainder is below the last subtraction.]

Ensure that they check their answers by reversing the thinking (e.g. make 7 groups of 4 and include the remainder of 2 to obtain 30).

Activity 8: Addresses achievement indicator 3 (no remainder).

Present a problem that involves a large 2-digit number divided by 2 having no remainder and where the answer is the number of groups formed (e.g. *Sally puts her 78 toys equally into groups of 2. How many groups of 2 did she make?*). Ask students to use a paper-and-pencil algorithm to figure out the answer and suggest that they think in terms of removing large numbers of groups of 2. Discuss their methods. Show and discuss method A with students. Ensure they realize using place value thinking helps make the multiplication easier.

Revisit 78 ÷ 2. Ask students to obtain the answer by removing the largest place value groups possible. Discuss their methods. Show and discuss method B with students. Lead them to understand that removing the largest place value groups possible (in this case 30 groups of 2) makes it faster to obtain the answer, and involves less writing.

Method A

2 | 78	
- 20	10
58	
- 20	10
38	
- 20	10
18	
- 10	5
8	
8	4
0	39

Method B

2 | 78	
- 60	30
18	
- 10	5
8	
- 8	4
0	39

Activity 9: Practice (no remainder).

Provide four 2-digit (mixed size) divided by 1-digit arithmetic questions having no remainders. Encourage students to remove the largest place value groups possible (method B). Ask for and discuss their solutions.

Activity 10: Practice (with remainder).

Provide four 2-digit (mixed size) divided by 1-digit arithmetic questions having remainders. Encourage students to remove the largest place value groups possible (method B). Ask for and discuss their solutions.

Note:

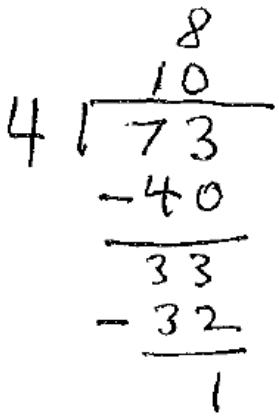
Students may appreciate a different writing style, one that looks a lot like what students' parents likely learned at school. This style might just bring "peace" in the family. It might even get the parents to understand what they learned years ago as a 'do this, don't ask why it works' method.

Activity 11: Addresses achievement indicator 3 (with and without remainder).

Present students with a 2-digit divided by division question having a remainder (e.g. 73 ÷ 4). Discuss writing styles with them. Refer to such situations as ways to write letters of the alphabet, ways to write numerals, and so on.

Show them a different writing style (see example) and discuss it as each group count and subtraction is written down. [The answer for the example is 18, remainder 1.]

Present students with four 2-digit ÷ 1-digit questions (two with remainders, two without). Ask students to use the new writing style for doing the division. Ask for and discuss solutions.



Note:

from this point on, students are free to choose their preferred writing style.

Activity 12: Addresses achievement indicator 3, 4, & 5 (with & without remainder).

- Present a problem that involves a 2-digit number divided by a 1-digit number where the answer concerns the number in each group (e.g. *Sally has 45 toys. She put them equally into 5 toy boxes. How toys are in each box?*). Discuss what is wanted (e.g. the number of toys in each box). Discuss whether the answer to 45 ÷ 5 would be the same if the question was the number of toys in each box or the number of boxes. Ask students to show the situation by putting 45 objects into groups of 5 and getting 9 groups as a result and by putting 45 objects into 5 groups and getting 9 in each group as a result. Lead students to understand that we can use the paper-and-pencil method to figure out the answer to either type of question (how many in each group or how many groups).
- Organize students into groups. Each group makes up one division problem for which the question concerns '<u>how many groups</u>' and one question which concerns '<u>how many in each group</u>'. One of the questions must involve a remainder; the other, no remainder. Each group creates an answer key for its two questions. Have the groups share and solve each others' problems. If there is a remainder, ask students to treat the remainder in a real way.
- Collect the answer keys, check to see they are correct, reproduce and distribute them to the groups to use for correcting the problems. Discuss the problems and solutions.

Activity 13: Practice

Provide a worksheet that has four 2-digit ÷ 1-digit division questions (not problems). Ask students to remove the largest place value groups possible when doing the division. Ask for and discuss answers.

Activity 14: Assessment of teaching.

Provide two division problems (problems, not arithmetic questions) where one problem involves figuring out how many groups are formed and other problem involves figuring out how many in each group. One problem involves a remainder; the other does not. For the remainder, ask students to treat it in a real way. [For example: *A van can carry 6 students. 27 students are going on a field trip by van. How many vans are needed?* [The realistic answer to this problem is you need 5 vans, not 4 vans with a remainder of 3.] Ask students to figure out the solutions to the problems using the paper-and-pencil method (the subtractive algorithm).

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

One example of partially well-designed worksheet follows.

More questions of each type are needed for a well-designed worksheet.

The MAINTAIN stage follows the sample worksheet.

Question 1.

You have 17 tickets to a school play.
You want to give 5 tickets to each friend.
How many friends can you give the tickets to? _____
How many tickets will be left over?

Question 2.

Alexa put 73 apples in bags of 6.
Mike put 46 apples in bags of 4.
Who had more apples left over? Show your work.

Question 3.

A canoe can hold 3 kids. How many canoes will 44 kids need? Show your work.

Question 4.

Anne reads 5 pages of her favourite book before bed every night.
She has 78 pages left to read. How many nights will it take her to finish her book?
Show your work.

Question 5.

What is the answer (and remainder, if any) to each division question? Try to obtain the answer by removing the most groups possible. Show your work.

37 ÷ 4

83 ÷ 5

91 ÷ 8

123 ÷ 6 (bonus)

MAINTAIN stage

Mini-task example

From time to time, present a simple word problem, sometimes with a reminder, sometimes without (see example). Ask students to solve the problem, using paper-and-pencil division method.

Example:

George has 56 toys to put into 7 boxes. He wants an equal number of toys in each box. How many toys go into each box?

Rich-task example

Ask students to solve the following problem.

George the dragon went grocery shopping the night before his birthday party. He wanted to have plenty of two-legged creatures to serve as the main meal for his guests. He caught 62 two-legged creatures in his favourite hunting spot near a village. He caught 27 two-legged creatures near a lake. He caught 3 two-legged creatures on the way home. George put all the two-legged creatures he caught into a cage. The next day George set 8 tables, one table for each guest. George tied up all the two-legged creatures he caught and placed an equal number on each table. George looked in the cage and noticed some creatures left over. He decided that he would eat these creatures before the guests arrived. How many two-legged creatures did George eat? Use division to figure out the answer. Use multiplication to figure out the answer. Did you get the same answer?

Comments.

This is a rich-task because it involves solving a complex problem.